Tenable strategy blocks and settled equilibria

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Abstract. When people interact in familiar settings, social conventions usually develop so that people tend to disregard alternatives outside the convention. For rational players to usually restrict attention to a block of conventional strategies, no player should prefer to deviate from the block when others are likely to act conventionally and rationally inside the block. We explore two set-valued concepts, coarsely and finely tenable blocks, that formalize this notion for finite normal-form games. We then identify settled equilibria, which are Nash equilibria with support in minimal tenable blocks. For a generic class of normal-form games, our coarse and fine concepts are equivalent, and yet they differ from standard solution concepts on open sets of games. We demonstrate the nature and power of the solutions by way of examples. Settled equilibria are closely related to persistent equilibria but are strictly more selective on an open set of simple games.

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JEL-codes: C70, C72, C73, D01, D02, D03.

1. Introduction

Schelling (1960) pointed out the importance and subtlety of pure coordination problems, that is, problems in which the participants have common interests but there are multiple ways to coordinate. Sometimes one of the solutions may be "salient" (Schelling, 1960). However, in many situations we must in practice rely on what

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Lewis (1969) calls precedent in order to solve coordination problems. If all participants know that a particular coordination problem has been solved in a particular way many times before, and this is common knowledge, then this may help them solve a current coordination problem.

More generally, consider a large population that plays familiar games, not necessarily coordination games, in historical and cultural contexts where individuals know how similar games have been played in the past. When people interact in such settings, social norms or conventions usually develop, specifying which actions or decision alternatives individuals are expected to, or should, consider.[1] Such informal institutions, norms or conventions develop over time and people tend to disregard alternatives that are physically available to them but fall outside the norm or convention. Arguably, this is a pervasive phenomenon in all societies. Social institutions are sustained in a larger natural interactive context by viewing such unconventional actions as "illegal" (Hurwicz, 2008). Conformity with social norms helps simplify people's decision-making and coordination. When people are generally expected to act rationally within the conventional norms, unconventional alternatives should not be advantageous.

The idea to embed a strategic interaction in a larger societal context is not new to game theory. Already in his Ph.D. thesis John Nash mentioned such a perspective, his so-called mass-action interpretation:

> "It is unnecessary to assume that the participants in a game have full knowledge of the total structure of the game, or the ability and inclination to go through any complex reasoning processes. But the participants are supposed to accumulate empirical information on the relative advantages of the various pure strategies at their disposal. ... To be more detailed, we assume that there is a population (in the sense of statistics) of participants for each position of the game. Let us also assume that the 'average playing' of the game involves n participants selected at random from the n populations, and that there is a stable average frequency with which each pure strategy is employed ... The assumptions we made in this 'mass action' interpretation lead to the conclusion that the mixed strategies representing the average behavior in each of the populations form an equilibrium." (Nash, 1950)

We here elaborate a theory which permits the endogenous formation of such conventions in finite games in normal form. A *block* in such a game is a non-empty set of pure strategies for each player role. We view a block as a potential norm or convention, a candidate for what strategies individuals are likely to seriously consider

[1] By a convention we mean a pattern of behavior that is customary, expected, and self-enforcing, see Lewis (1969) and Young (1993,1998).

when called upon to play the game in their player role. The associated *block game* is the restricted game in which all players are confined to their block strategies. A robustness requirement on a block to be a potential convention is that nobody should be able to do better by choosing a strategy outside the block when others are very likely to use strategies in the block. We call blocks that satisfy a slightly weaker robustness requirement *coarsely tenable*. The weakening is that this robustness should hold at least when the overall population play is at a population-level equilibrium. A *coarsely settled equilibrium* is any Nash equilibrium (of the whole game) with support in a minimal coarsely tenable block. People tend to forget or disregard unused strategies, and minimality allows players to disregard as many unused pure strategies as possible. This simplifies the convention and saves on players' cognitive costs.[2] Minimality can also be viewed as requirement of "internal stability"; that no part of a conventional block can eliminated without losing its stability.[3] So in a sense, minimal coarsely tenable blocks exhibit both a form of "internal" and "external" stability.

Coarse tenability imposes no constraint on "non-conventional" players, those who consider other strategy subsets than the conventional block. In other words, if "player types" specify what strategy subsets players consider when choosing their strategy, robustness is required under *any* probability distribution over "player types" that assigns sufficient probability on the conventional "types". Given the traditional emphasis that game theory places on rationality, a relevant restriction on such type distributions would be that, among the unconventional types, "more rational" types should be much more prevalent than "less rational" types. Arguably, a "more rational" decision maker considers more options before making a decision.[4] We hence focus on type distributions that assign much less probability to one player type than another if the second type only considers a (strict) subset of the strategies considered by the first. We call a block *finely tenable* if there is no strategy outside the block that would be a better reply for such "rationality biased" type distributions. Since we impose additional assumptions on what individuals are likely to consider when not conventional, every coarsely tenable block is, *a fortiori*, also finely tenable. Thus, there are more finely than coarsely tenable blocks to choose from as potential

[2] See e.g. Halpern and Pass (2009) and their references.

[3] See the discussion of stable sets in von Neumann and Morgernstern (1944), sections 4.1 and 4.5-4.7, where the authors claim that "... the rules of rational behavior must provide definitely for the possibility of irrational conduct on the part of others" (4.1.2) and ". it appears that the sets of imputations which we are considering correspond to the 'standards of behavior' connected with a social organization" (4.6.1), and "Thus our solutions S correspond to such 'standards of behavior' as have an inner stability: once they are generally accepted they overrule everything else and no part of them can be overruled within the limits of the accepted standards." (4.6.2).

[4] See Remark X for a weakinging of this requirement to a requirement that this set-wise monotonicity should hold only if the superset contains "strategically relevant" alternatives absent from the subset.

conventions.

Any *absorbing* block (Kalai and Samet, 1984) or any *curb* block (Basu and Weibull, 1991) meets these robustness requirements. However, such blocks are sometimes very large, and, moreover, may depend on aspects that arguably should be regarded as strategically inessential. The mentioned block properties are nested: curb implies absorbing, absorbing implies coarsely tenable, and coarsely tenable implies finely tenable. As shown in Ritzberger and Weibull (1995), every curb block contains a hyperstable set and hence the support of a proper equilibrium. Moreover, proper equilibria are known to induce a (realization equivalent) sequential equilibrium in every extensive-form with the given normal form (van Damme, 1981). In other words, the most stringent block property is consistent with equilibrium theory in a very refined form. We here show that even the least stringent block property, that of fine tenability, has this property. We accordingly define a *finely settled* equilibrium as any proper equilibrium with support in a minimal finely tenable block. A *fully settled equilibrium* is a Nash equilibrium that is both coarsely and finely settled. By construction, every finite game has at least one minimal coarsely tenable block that (weakly) contains at least one finely tenable block. Hence, every finite game admits at least one fully settled equilibrium.

While the notions of coarse and fine tenability in general differ, we show that they coincide for generic normal-form games. By contrast, while Nash equilibria are generically perfect and proper, this is not true for coarsely and finely settled equilibria. The latter, while being generically identical with each other, constitute a strict subset of the Nash equilibria in an open set of normal-form games. They also constitute a distinct subset from the persistent equilibria (Kalai and Samet, 1984) in an open set of normal-form games.

Before entering the analysis, let us briefly illustrate the above reasoning by means of a simple coordination game,

$$\begin{array}{ccc} & L & R \\ L & \alpha, \beta & 0, 0 \\ R & 0, 0 & \gamma, \delta \end{array} \tag{1}$$

where $\alpha, \beta, \gamma, \delta > 0$. Such a game has two pure and strict and one mixed Nash equilibrium. All three are proper equilibria that, when viewed as singleton sets, are strategically stable in the sense of Kohlberg and Mertens (1986). If the game is played only once by rational players, in the absence of a cultural, historical or social context, the mixed equilibrium may be a reasonable prediction. Indeed, in these games any strategy profile is rationalizable and thus compatible with common knowledge of the game and the players' rationality (Bernheim, 1984; Pearce, 1984; Brandenburger and Dekel, 1987; Tan and Werlang, 1988). Hence, when played once, common knowledge of the game and the players' rationality has no predictive power.

However, if such a game is often played in culturally familiar settings, the mixed equilibrium appears unlikely, as do many other strategy profiles. One would expect individuals to develop an understanding that coordinates their expectations at one of the strict equilibria. This intuition is captured by the solution concepts developed here and also by persistent equilibrium. However, in other games, our solutions differ from persistence and this is true even under arguably minor elaborations of the game (1), such as when one or both of the zero-payoff outcomes is replaced by a zero-sum game; then persistence accepts the mixed equilibrium while the present approach still rejects it. We also show that our approach rejects the mixed equilibrium strategy in game (1) even if it is added as a third pure strategy.

The rest of the paper is organized as follows. Basic notation and definitions are given in Section 2. Our model of consideration sets is developed in Section 3. Coarsely tenable blocks and coarsely settled equilibria are introduced in Section 4 and finely tenable blocks and finely settled equilibria are given in Section 5. In Section 6 we shows that coarsely and finely tenable blocks generically coincide. The nature and power of tenability and settledness is demonstrated in examples throughout the text. Section 7 provides additional examples, and Section 8 concludes.

2. Preliminaries

We consider finite normal-form games $G = \langle N, S, u\rangle$, where $N = \{1, \ldots, n\}$ is the set of players, $S = \times_{i\in N} S_i$ is the non-empty and finite set of pure-strategy profiles, $u : S \to \mathbb{R}^n$ is the combined payoff function, where $u_i(s) \in \mathbb{R}$ is i's payoff under pure-strategy profile s. Let m_i be the number of elements of S_i and let $\Delta(S_i)$ denote the set of mixed strategies available to player i:

$$\Delta(S_i) = \left\{ \sigma_i \in \mathbb{R}_+^{m_i} : \sum_{s_i \in S_i} \sigma_i(s_i) = 1 \right\}.$$

A strategy $\sigma_i \in \Delta(S_i)$ is *totally mixed* if it assigns positive probability to all pure strategies. Write $\Delta^o(S_i)$ for this subset. Likewise, a strategy profile is totally mixed if all strategies are totally mixed. Let $M(S) = \times_{i\in N}\Delta(S_i)$ denote the set of mixed-strategy profiles on S and let $M^o(S) = \times_{i\in N}\Delta^o(S_i)$. We extend the domain of each payoff function u_i in the usual way from S to $M(S)$ by

$$u_i(\sigma) = \sum_{s\in S} \left[\Pi_{j\in N}\sigma_j(s_j)\right] \cdot u_i(s).$$

We use $u_i(s_{-i}, s_i')$ to denote the payoff that player i obtains from pure strategy $s_i' \in S_i$ when everyone else plays according to $s \in S$, and likewise for mixed strategies. Likewise, let $u_i(\sigma_{-i}, [s_i])$ be the (expected) payoff that player i obtains from pure strategy $s_i \in S_i$ when everyone else plays according to $\sigma \in M(S)$. Two pure strategies, $s_i', s_i'' \in S_i$, are *payoff equivalent* if $u_i(s_{-i}, s_i') = u_i(s_{-i}, s_i'')$ for all $s \in S$ and all players $i \in N$. A *purely reduced* normal form game is a game in which no pure strategies are payoff equivalent.[5] Two pure strategies, $s_i', s_i'' \in S_i$, are *payoff equivalent for player i* if $u_i(s_{-i}, s_i') = u_i(s_{-i}, s_i'')$ for all $s \in S$. A pure strategy $s_i \in S_i$ is *weakly dominated* if there exists a $\sigma_i' \in \Delta(S_i)$ such that $u_i(\sigma_{-i}, \sigma_i') \geq u_i(\sigma_{-i}, [s_i])$ for all $\sigma \in M(S)$ with strict inequality for some $\sigma \in M(S)$. A *Nash equilibrium* is any strategy profile $\sigma \in M(S)$ such that

$$u_i(\sigma_{-i}, [s_i]) < \max_{r_i \in S_i} u_i(\sigma_{-i}, [r_i]) \quad \Rightarrow \quad \sigma_i(s_i) = 0.$$

A Nash equilibrium is *strict* if any unilateral deviation incurs a payoff loss.

Definition 1 [Myerson, 1978]**.** *For any $\varepsilon > 0$, a strategy profile $\sigma \in M^o(S)$ is ε-**proper** if*

$$u_i(\sigma_{-i}, [s_i]) < u_i(\sigma_{-i}, [r_i]) \quad \Rightarrow \quad \sigma_i(s_i) \leq \varepsilon \cdot \sigma_i(r_i).$$

*A **proper equilibrium** is any limit of ε-proper strategy profiles as $\varepsilon \to 0$.*

The proper equilibria constitute a non-empty subset of the Nash equilibria. We next turn to the concepts of a persistent retract and a persistent equilibrium. Every finite game has a persistent retract and a persistent equilibrium.

Definition 2 [Kalai and Samet, 1984]**.** *A **retract** is any set $X = \times_{i \in N} X_i$ such that $\varnothing \neq X_i \subseteq \Delta(S_i)$ is closed and convex $\forall i \in N$. A retract X is **absorbing** if it has a neighborhood $U \subseteq M(S)$ such that for all $\sigma' \in U$:*

$$\max_{\sigma_i \in X_i} u_i(\sigma'_{-i}, \sigma_i) = \max_{s_i \in S_i} u_i(\sigma'_{-i}, [s_i]) \qquad \forall i \in N.$$

*A **persistent retract** is any minimal absorbing retract. A **persistent equilibrium** is any Nash equilibrium belonging to a persistent set.*

We will use the following terminology and notation: a *block* is any set $T = \times_{i \in N} T_i$ such that $\varnothing \neq T_i \subseteq S_i \ \forall i \in N$. The associated *block game* is the game $G_T = \langle N, T, u \rangle$ (with u restricted to T). We embed its mixed strategies in the full strategy space of the game G: $M(T) = \{\sigma \in M(S) : \sigma_i(s_i) = 0 \ \forall s_i \notin T_i, \forall i \in N\}$. If T is a block, then clearly $M(T)$ is a retract. By a slight abuse of language, we will call a block T absorbing if $M(T)$ is absorbing. A strategy profile σ has *support in a block T* if $\sigma_i(s_i) = 0$ for all players $i \in N$ and strategies $s_i \notin T_i$. Write $\sigma(T)$ for the probability that a mixed-strategy profile $\sigma \in M(S)$ puts on a block: $\sigma(T) = \sum_{s \in T} [\Pi_{i \in N} \sigma_i(s_i)]$. Thus $\sigma \in M(T)$ iff $\sigma(T) = 1$.

[5] This is also called the *semi-reduced* normal form, see e.g. van Damme (1991).

Definition 3. *A* ***Nash equilibrium of a block game*** G_T *is any strategy profile* $\sigma \in M(T)$ *such that*

$$\sigma_i(s_i) > 0 \quad \Rightarrow \quad s_i \in \arg\max_{t_i \in T_i} u_i(\sigma_{-i}, [t_i]).$$

Clearly every block game has at least one Nash equilibrium.

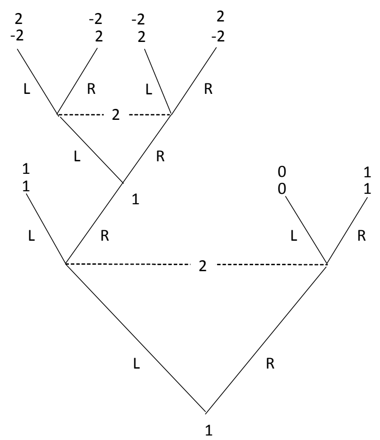
Definition 4 [Basu and Weibull, 1991]**.** *A block T is* ***curb*** *("closed under rational behavior") if*

$$\arg\max_{s_i \in S_i} u_i(\sigma_{-i}, [s_i]) \subseteq T_i$$

for every strategy profile $\sigma \in M(T)$ and every player $i \in N$.

Every finite game has a minimal curb block. As noted in Ritzberger and Weibull (1995), every curb block is absorbing.[6] The game in the introduction has two minimal curb blocks, the supports of its two strict equilibria. Hence, the mixed equilibrium is not persistent. However, in a slight elaboration of that game, the only absorbing block is the full pure-strategy space, so persistence then looses all its cutting power on the set of Nash equilibria.

Example 1. *Consider the extensive-form game*



This is an elaboration of game (1) for $\alpha = \beta = \gamma = \delta = 1$, where the added subgame is a zero-sum game with value zero. Hence, backward induction requires the players to attach value zero to the subgame, which arguably renders the elaborated game "strategically equivalent" with the original game.

[6]To see this, suppose that T is curb. By continuity of payoff functions and the finiteness of the game, there exists a a neighborhood U of $M(T) \subseteq M(S)$ such that all mixed-strategy profiles in U have all best replies in the block.

The purely reduced normal-form representation of the elaboration is

Game 1:

	L	RL	RR
LL	$1,1$	$2,-2$	$-2,2$
LR	$1,1$	$-2,2$	$2,-2$
R	$0,0$	$1,1$	$1,1$

This game has three Nash equilibrium components: A, B and $C = \{\sigma^m\}$, where A consists of all strategy profiles of the form $\sigma = (p[LL] + (1-p)[LR]\ ,\ [L])$ for $1/4 \leq p \leq 3/4$, B consists of all strategy profiles of the form $\sigma = ([R]\ ,\ q[RL] + (1-q)[RR])$ for $1/4 \leq q \leq 3/4$, and

$$\sigma^m = (\ \frac{1}{2}[R] + \frac{1}{4}[LL] + \frac{1}{4}[LR])\ ,\ \frac{1}{2}[L] + \frac{1}{4}[RL] + \frac{1}{4}[RR]).$$

There are three proper equilibria: $\sigma^a = (\frac{1}{2}[LL] + \frac{1}{2}[LR], [L]) \in A$, $\sigma^b = ([R], \frac{1}{2}[RL] + \frac{1}{2}[RR]) \in B$, and σ^m. The only absorbing retract is $M(S)$, so S is the only curb set and all Nash equilibria are persistent. The last conclusion is valid for all games (1) with $(\alpha, \beta, \gamma, \delta)$ in an open set containing $(1,1,1,1)$.

The mixed equilibrium σ^m in this example would arguably be non-robust as a convention, since individuals would presumably learn to avoid the zero-sum subgame and instead be likely to end up in one of the equilibrium components A or B. The solution concepts to be developed below formalize such intuitions.

Remark 1. *Other related ideas in the recent literature are so-called prep sets (Voorneveld 2004, 2005) and p-best response sets (Tercieux, 2006 a,b). A prep set (or preparation) is a block T that contains at least one best reply for each player to every mixed strategy on the block. Every pure Nash equilibrium (viewed as a singleton block) is thus a prep set and every curb set is a prep set. Voorneveld (2004) shows that minimal prep sets generically coincide with minimal curb sets and Voorneveld (2005) establishes that prep sets also generically coincide with persistent retracts (T being a prep set and $M(T)$ an absorbing retract). Tercieux (2006a) defines a p-best response set as a block that contains all best replies to all beliefs that put at least probability p on the block, where beliefs are not constrained to treat other players' strategy choices as statistically independent (a constraint we here impose). Tercieux (2006b) weakens the requirement "all best replies" to "some best reply," and calls the first notion strict p-best response sets. For all finite two-player games: (a) any strict p-best response set with $p < 1$ is curb, and every curb set is a strict p-best response set for some $p < 1$ (see Lemma 2 in Ritzberger and Weibull, 1995), and (b) if a block T is a (weak) p-best response set with $p < 1$, then $M(T)$ is an absorbing retract, and if $M(T)$ is an absorbing retract, then T is a (weak) p-best response set for some $p < 1$.*

It follows that T is a minimal (weak) p-best response set for some $p < 1$ if and only if $X = M(T)$ is a persistent set. Another related idea is the refined best-response correspondence in Balkenborg, Hofbauer, and Kuzmics (2013,2014). Their correspondence shares many properties with the usual best-response correspondence—such as being upper hemi-continuous, closed- and convex-valued—but generically differs in games with more than two players. However, in two-player normal form games it generically coincides with the usual best-response correspondence. Hence, their solutions generically differ from ours.

3. CONSIDERATION-SET GAMES

We proceed to construct a framework within which one can make precise the idea that conventions or norms should be such that when people are generally expected to act rationally within the convention or norm, unconventional alternatives should not be advantageous. We do this in terms of a situation in which individuals are very likely to consider only the strategies in some conventional block, but allowing for the possibility that some individuals may also consider strategies outside the block. An individual's effective strategy set, to be called his or her *consideration set*, a non-empty subset of the full strategy set, will be treated as his or her *type* in a game of incomplete information where types are private information.[7]

More precisely, let $G = \langle N, S, u \rangle$ be a finite game. As in Nash's mass action interpretation, let there for each player role $i \in N$ be a large population of individuals who are now and then randomly called upon to play the game G in that player role. Let the type space for each player role $i \in N$ be $\Theta_i = \mathcal{C}(S_i)$, the collection of non-empty subsets $C_i \subseteq S_i$. Let μ_i be any probability distribution over $\mathcal{C}(S_i)$, where $\mu_i(C_i) \in [0, 1]$ is the probability that the individual drawn to play in role i will be of type $\theta_i = C_i$, that is, have C_i as his or her consideration set. These random draws of types, one draw for each player population, are statistically independent. A vector $\mu = (\mu_1, .., \mu_n) \in \times_{i \in N} \Delta(\mathcal{C}(S_i))$ is thus a *type distribution*, where the probability that any given block $T = \times_{i \in N} T_i$ will be the actual consideration block is the product of the probabilities for each player role; $\mu_1(T_1) \cdot .. \cdot \mu_n(T_n)$. With some abuse of notation, we will write $\mu(T)$ for this product probability.

Each type distribution μ defines a game $G^\mu = \langle N, F, u^\mu \rangle$ of incomplete information in which a pure strategy for each player role $i \in N$ is a function $f_i : \mathcal{C}(S_i) \to S_i$ such that $f_i(C_i) \in C_i$ for all $C_i \in \mathcal{C}(S_i)$. In other words, a pure strategy f_i prescribes for each type $\theta_i \in \Theta_i$ a pure strategy in the type's consideration set C_i. Let F_i be the set of such functions and write $F = \times_{i \in N} F_i$. Each pure-strategy profile $f \in F$ induces

[7]The term "consideration set" is borrowed from management science and marketing. The basic idea is that decision-makers may not consider all choices available to them. The term originates with Wright and Barbour (1977). For recent contributions to this literature, see Manzini and Mariotti (2007, 2013), Salant and Rubinstein (2008), and Eliaz and Spiegler (2011).

a mixed-strategy profile $\sigma^{f,\mu} \in M(S)$ in G, where the probability that player $i \in N$ will use pure strategy $s_i \in S_i$ is

$$\sigma_i^{f,\mu}(s_i) = \sum_{C_i \in \mathcal{C}(S_i)} \mu_i(C_i) \cdot \mathbf{1}_{f_i(C_i)=s_i}.$$

The resulting expected payoff to each player i is $u_i^\mu(f) = u_i\left(\sigma^{f,\mu}\right)$. This defines the payoff functions $u_i^\mu : F \to \mathbb{R}$ for all players $i \in N$ in G^μ. The *consideration-set game* G^μ, so defined, is finite. Payoffs to mixed-strategy profiles can be defined in the usual way. By Nash's existence theorem, each consideration-set game G^μ has at least one Nash equilibrium in pure or mixed strategies.[8]

For any mixed-strategy profile $\tau \in M(F)$, player role $i \in N$ and strategy subset $C_i \in \mathcal{C}(S_i)$, let $\tau_{i|C_i} \in \Delta(S_i)$ be the conditional probability distribution over the strategy set S_i, given that $\theta_i = C_i$ is i's type (in particular, $\tau_{i|C_i}(s_i) = 0 \ \forall s_i \notin C_i$). When a mixed-strategy profile $\tau \in M(F)$ is played in G^μ, pure strategy $s_i \in S_i$ will be used with probability

$$\tau_i^\mu(s_i) = \sum_{C_i \in \mathcal{C}(S_i)} \mu_i(C_i) \cdot \tau_{i|C_i}(s_i). \tag{2}$$

This defines the mixed-strategy profile $\tau^\mu \in M(S)$ induced by τ in the underlying game G. We will sometimes refer to τ^μ as the *projection* of $\tau \in M(F)$ to $M(S)$ under the type distribution μ.

A strategy profile $\tau \in M(F)$ is a Nash equilibrium of G^μ if and only if for all player roles $i \in N$ and consideration sets $C_i \in \mathcal{C}(S_i)$,

$$\mu_i(C_i) > 0 \quad \Rightarrow \quad u_i(\tau_{-i}^\mu, \tau_{i|C_i}) = \max_{s_i \in C_i} u_i(\tau_{-i}^\mu, [s_i]). \tag{3}$$

It is easily verified that Nash equilibria of G^μ converge to Nash equilibria of G_T as $\mu(T) \to 1$.

4. COARSELY TENABLE BLOCKS AND COARSELY SETTLED EQUILIBRIA

Let $G = \langle N, S, u \rangle$ be any finite game and let T be any block, interpreted as a potential convention. Call an individual in player population $i \in N$ *conventional* if his or her type is $\theta_i = T_i$. The following definition formalizes an arguably weak robustness requirement on such a convention, namely, that if the type distribution μ is such that individuals are very likely to be conventional, and if their average play is a Nash

[8] A special case of this set-up is when $\mu(S) = 1$. Then G^μ is effectively the same as G; the probability is then one that all players will consider all pure strategies at their disposal in G.

equilibrium of the associated game of incomplete information, G^{μ}, then nobody could do better by choosing a strategy outside the block.[9]

Definition 5. *A block T is **coarsely tenable** if there exists an $\varepsilon \in (0,1)$ such that*

$$\max_{t_i \in T_i} u_i(\tau^{\mu}_{-i}, [t_i]) = \max_{s_i \in S_i} u_i(\tau^{\mu}_{-i}, [s_i]) \quad \forall i \in N \tag{4}$$

for every type distribution μ with $\mu_i(T_i) > 1 - \varepsilon$ $\forall i \in N$ and every Nash equilibrium τ of the associated game G^{μ}.

By (3), the full block $T = S$ is coarsely tenable in this sense. Also a singleton block that is the support of any pure strict equilibrium is coarsely tenable, and so is any curb block and any absorbing block. To see why the last claim holds, let T be an absorbing block. By definition, there then exists an $\varepsilon \in (0,1)$ such that

$$\max_{t_i \in T_i} u_i(\sigma_{-i}, [t_i]) = \max_{s_i \in S_i} u_i(\sigma_{-i}, [s_i]) \quad \forall i \in N$$

if $\sigma \in M(S)$ is such that $\sigma_i(T_i) > 1 - \varepsilon$ for all players $i \in N$. Let $\tau \in M(F)$ be a Nash equilibrium of any consideration-set game G^{μ} such that $\mu_i(T_i) > 1 - \varepsilon$ for this ε and for all players i. Then $\tau^{\mu}_i(T_i) > 1 - \varepsilon$ for all players $i \in N$.[10] Thus (4) holds.

We also note that the equilibria of the block game associated with a coarsely tenable block coincide with the equilibria of the original game that have support in the block. In other words, oblivion of strategies outside the block then comes at no cost. In sum:

Proposition 1. *Every absorbing block is coarsely tenable. If a block T is coarsely tenable, then the Nash equilibria of the block game G_T are precisely the Nash equilibria of G that have support in T.*

Proof: To prove the last claim, let T be any block in any finite game G. First, if $\sigma \in M(T)$ is a Nash equilibrium of G, it is, *a fortiori*, a Nash equilibrium of G_T. Secondly, suppose that $\sigma \in M(T)$ is a Nash equilibrium of G_T. Let $\mu(T) = 1$ and let

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[9] We do not require or presume Nash equilibrium play in the consideration-set game G^{μ}, only that if this were the case, then—at least then—no player in G should have a better strategy outside the block.

[10] By definition,

$$\tau^{\mu}_i(T_i) = \sum_{t_i \in T_i} \tau^{\mu}_i(t_i) = \sum_{t_i \in T_i} \sum_{C_i \in \mathcal{C}(S_i)} \mu_i(C_i) \cdot \tau_{i|C_i}(t_i) \geq \mu_i(T_i) \cdot \sum_{t_i \in T_i} \tau_{i|T_i}(t_i) = \mu_i(T_i) > 1 - \varepsilon.$$

$\tau \in M(F)$ be such that $\tau_{i|T_i} = \sigma_i$. Then $\tau^\mu = \sigma$, and by (3), τ is a Nash equilibrium of G^μ. If T is coarsely tenable:

$$u_i(\sigma) = u_i(\tau^\mu_{-i}, \tau_{i|T_i}) = \max_{t_i \in T_i} u_i(\sigma_{-i}, [t_i]) = \max_{s_i \in S_i} u_i(\tau^\mu_{-i}, [s_i]) = \max_{s_i \in S_i} u_i(\sigma_{-i}, [s_i]) \quad \forall i \in N.$$

Q.E.D.

Kalai and Samet (1984) show that elimination of weakly dominated strategies and/or payoff-equivalent strategies from the full strategy space $M(S)$ results in an absorbing retract.[11] Since absorbing blocks are coarsely tenable, the same qualitative conclusions hold for coarsely tenable blocks. More precisely, for each player i in G, let $T_i \subseteq S_i$ be such that every pure strategy not in T_i is weakly dominated by some mixed strategy with support in T_i. Then T is coarsely tenable, since each player i will have some (globally) best reply in T_i to the projection τ^μ of any mixed-strategy profile $\tau \in M(F)$ in any consideration-set game G^μ. Likewise, for each player i, let $T_i \subseteq S_i$ be such that for every pure strategy not in T_i there exists a strategy in T_i that is payoff-equivalent for player i. Then T is coarsely tenable.

Conventions tend to simplify the interaction at hand by excluding as many strategies as possible. This suggests that minimal coarsely tenable blocks are particularly relevant for prediction.[12] The games we study are finite and hence admit at least one such block. The following definition formalizes an equilibrium notion that combines the (simplicity) requirement of minimality of the set of conventional strategies with the (rationality) requirement that individuals should not be able to benefit by using unconventional strategies when others are likely to use conventional strategies.

Definition 6. *A* ***coarsely settled equilibrium*** *is any Nash equilibrium of G that has support in some minimal coarsely tenable block T.*

Evidently, any pure strict equilibrium is coarsely settled. By contrast, the mixed equilibrium in game (1) is not coarsely settled, since it does not have support in a minimal coarsely tenable block.[13] In that example, also the notion of persistent equilibrium rejects the mixed equilibrium. By contrast, in the elaborated version of this game, Game 1 in Example 1, the totally mixed Nash equilibrium was seen to be persistent. However, it is not coarsely settled. Game 1 has two minimal coarsely tenable blocks, associated with each of the two continuum Nash equilibrium components, A and B. These blocks are $T^A = \{LL, LR\} \times S_2$ and $T^B = S_1 \times \{RL, RR\}$. The coarsely settled equilibria of Game 1 are the Nash equilibria in these two components.

[11] Here payoff equivalent can be interpreted in the weak sense of payoff equivalence for the player in question (see Section 2).

[12] A coarsely tenable block is *minimal* if it does not properly contain any coarsely tenable block.

[13] In Section 7 we show that this conclusion holds also when the mixed equilibrium is represented as a pair of pure strategies added to the game.

5. Finely tenable blocks and finely settled equilibria

Imposing some structure on the type distributions in the consideration-set games, beyond placing high probabilities on the conventional types, could allow for smaller blocks—a finer block structure. The following definition formalizes the notion that (a) individuals are very likely to be of the conventional types for the block (as under coarse tenability), (b) all types have positive probability, and (c) unconventional types (those with other consideration sets than those constituting the block) are much more likely to have larger than smaller consideration sets (in terms of set inclusion). In particular, the most likely among the unconventional types is the "standard" type of player in game theory, the one who considers all strategies available in his or her player role.

Definition 7. *For any block T and any $\varepsilon \in (0,1)$, a type distribution μ is **ε-proper on** T if*

$$\begin{cases} (a) & \mu_i(T_i) > 1 - \varepsilon \\ (b) & \mu_i(C_i) > 0 \quad \forall C_i \in \mathcal{C}(S_i) \\ (c) & T_i \neq C_i \subset D_i \quad \Rightarrow \quad \mu_i(C_i) \leq \varepsilon \cdot \mu_i(D_i) \end{cases}$$

for every player $i \in N$.

The following remark shows that a type distribution has this property if unconventional individuals' inattention to individual pure strategies are statistically independent.

Remark 2. *Let $G = \langle N, S, u \rangle$ be a finite game and let T be a block, interpreted as a potential convention. For all players $i \in N$ and all consideration sets C_i other than T_i, let*

$$\mu_i(C_i) = \varepsilon \cdot \Pi_{s_i \in C_i}(1 - \delta_i(s_i)) \cdot \Pi_{s_i \notin C_i}\delta_i(s_i) \tag{5}$$

(with the last product defined as unity in case $C_i = S_i$). This can be interpreted as follows. For each player role $i \in N$ in the game there is a large population of individuals who are now and then called to play the game, just as in Nash's mass-action interpretation. The fraction $1 - \varepsilon$ of each player population are conventional; their consideration sets are those that define the block. Among the unconventional individuals, who make up the population fraction $\varepsilon \in (0,1)$, each pure strategy $s_i \in S_i$ is ignored with some probability $\delta_i(s_i) \in (0,1)$, and these are statistically independent events for all pure strategies and individuals, hence the formula (5).[14]

[14] If an individual would in this way ignore all pure strategies in his player role, then he would "wake up" and consider the conventional set. This follows from (5):

$$\mu_i(T_i) = (1 - \varepsilon) + \varepsilon \cdot [\Pi_{s_i \in T_i}(1 - \delta_i(s_i)) \cdot \Pi_{s_i \notin T_i}\delta_i(s_i) + \Pi_{s_i \in S_i}\delta_i(s_i)]$$

Such a type distribution μ is ε-proper on T if all probabilities $\delta_i(s_i)$ are sufficiently small. To see this, let $\|\delta_i\| = \max_{s_i \in S_i} \delta_i(s_i)$. Clearly $\mu_i(T_i) > 1 - \varepsilon$ and $\mu_i(C_i) > 0$ for all $C_i \in \mathcal{C}(S_i)$. Suppose that $C_i, D_i \in \mathcal{C}(S_i)$, $C_i \subset D_i$ and $C_i \neq T_i$. Then

$$\mu_i(C_i) \leq \mu_i(D_i) \cdot \prod_{s_i \in D_i \setminus C_i} \frac{\delta_i(s_i)}{1 - \delta_i(s_i)} \leq \frac{\|\delta_i\|}{1 - \|\delta_i\|} \cdot \mu_i(D_i).$$

The factor in front of $\mu_i(D_i)$ is less than ε if $\|\delta_i\| < \varepsilon/(1+\varepsilon)$.[15]

By requiring robustness only to type distributions that are ε-proper on the block in question, one obtains the following weaker block property:

Definition 8. *A block T is* ***finely tenable*** *in $G = \langle N, S, u \rangle$ if there exists an $\varepsilon \in (0,1)$ such that*

$$\max_{t_i \in T_i} u_i(\tau^\mu_{-i}, [t_i]) = \max_{s_i \in S_i} u_i(\tau^\mu_{-i}, [s_i]) \quad \forall i \in N$$

holds for every type distribution μ that is ε-proper on T and every Nash equilibrium τ of G^μ.

Since every coarsely tenable block is, *a fortiori*, also finely tenable, there are, in general, more finely than coarsely tenable blocks. In particular, minimal finely tenable blocks may be smaller than the minimal coarsely tenable blocks.

Example 1 continued. *In each of the two minimal coarsely tenable blocks in Game 1 in Example 1, there are pure strategies not used in any block equilibrium. Player 2 uses only pure strategy L in all block equilibria in T^A and player 1 uses only pure strategy R in all block equilibria in T^B. Nevertheless, all 2's strategies need to be included in T^A and all 1's strategies in T^B, since otherwise there will be block equilibria with a better reply outside the block, and this would destabilize the block. In particular, the subblock $T^* = \{LL, LR\} \times \{L\}$ of the coarsely tenable block T^A is not coarsely tenable. However, it is finely tenable. To see this, let $\varepsilon \in (0,1)$ and let μ be any ε-proper type distribution on T^*. Consider any Nash equilibrium $\tau \in M(F)$ in the associated consideration-set game G^μ. Then $\tau^\mu_1(LL) = \tau^\mu_1(LR)$. For suppose that $\tau^\mu_1(LL) > \tau^\mu_1(LR)$. By (2) we then have, for ε sufficiently small, $\tau^\mu_2(RL) = \mu_2(\{RL\})$ and $\tau^\mu_2(RR) \geq \mu_2(\{RL, RR\})$. Since μ is ε-proper on T^*, we also have*

$$\mu_2(\{RL\}) \leq \varepsilon \cdot \mu_2(\{RL, RR\}),$$

so $\tau^\mu_2(RL) < \varepsilon \cdot \tau^\mu_2(RR)$. Then LR is a best reply for player 1, and $\tau^\mu_1(LL) < \tau^\mu_1(LR)$, a contradiction. By the same token, $\tau^\mu_1(LL) > \tau^\mu_1(LR)$ is not possible. A

[15] See Manzini and Mariotti (2013) for decision-theoretic foundations for such statistically independent inattention, and for relations with random-utility models.

similar argument establishes $\tau_2^\mu(RL) = \tau_2^\mu(RR)$. These two equations together imply that each player i has a best reply to τ^μ in T_i^*, that is, (4) holds and T^* is finely tenable.

When a block is finely tenable, the projection of any Nash equilibrium in any consideration-set game G^μ where the type distribution μ is ε-proper on T, constitutes an ε-proper strategy profile in the original game G.

Proposition 2. *Let T be a finely tenable block and let ε be as in Definition 8. If μ is any type distribution that is ε-proper on T, and if $\tau \in M(F)$ is a Nash equilibrium of G^μ, then $\tau^\mu \in M(S)$ is an ε-proper strategy profile in G.*

Proof: To show that τ^μ is an ε-proper strategy profile in G we first note that since each $C_i \in \mathcal{C}(S_i)$ has positive probability of being the consideration set under μ, $\tau_i(f_i) = 0$ for all pure strategies $f_i \in F_i$ such that $f_i(C_i) \notin \arg\max_{s_i \in C_i} u_i(\tau_{-i}^\mu, [s_i])$ for some $C_i \in \mathcal{C}(S_i)$. Secondly, let $r_i, s_i \in S_i$ be such that $u_i(\tau_{-i}^\mu, [r_i]) < u_i(\tau_{-i}^\mu, [s_i])$ and let $\mathcal{R}_i \subseteq \mathcal{C}(S_i)$ be the collection of sets $C_i \in \mathcal{C}(S_i)$ such that

$$r_i \in \arg\max_{c_i \in C_i} u_i(\tau_{-i}^\mu, [c_i]).$$

Clearly $C_i \in \mathcal{R}_i \Rightarrow s_i \notin C_i$. Moreover, $T_i \notin \mathcal{R}_i$, since T is finely tenable and thus contains a pure best reply to τ^μ. For each $C_i \in \mathcal{R}_i$:

(i) $\{s_i\} = \arg\max_{s_i' \in C_i \cup \{s_i\}} u_i(\tau_{-i}^\mu, [s_i'])$,

(ii) $\mu_i(C_i) \leq \varepsilon \cdot \mu_i(C_i \cup \{s_i\})$ and

(iii) $\sum_{C_i \in \mathcal{R}_i} \mu_i(C_i \cup \{s_i\}) \leq \tau_i^\mu(s_i)$

(where (iii) follows from the fact that, in equilibrium, pure strategy s_i is necessarily used when it is the unique best reply within the consideration set at hand). Hence,

$$\tau_i^\mu(r_i) \leq \sum_{C_i \in \mathcal{R}_i} \mu_i(C_i) \leq \varepsilon \cdot \sum_{C_i \in \mathcal{R}_i} \mu_i(C_i \cup \{s_i\}) \leq \varepsilon \cdot \tau_i^\mu(s_i).$$

(where the first inequality follows from the fact that, in equilibrium, pure strategy r_i is not used when it is not a best reply within the consideration set at hand). This establishes that $\tau^\mu \in M^o(S)$ is an ε-proper strategy profile in G. **Q.E.D.**

Remark 3. *The above proof holds also for weaker versions of fine tenability. One such version is obtained when the hypothesis in condition (c) in Definition 7 is strengthened to also require that D_i contains a strategy that is "strategically relevant" in the sense of being a strictly better reply to some strategy profile. Formally: replace (c) by the condition (c') that if $T_i \neq C_i \subset D_i$ and $\max_{s_i \in C_i} u_i(\sigma_{-i}, [s_i]) < \max_{s_i \in D_i} u_i(\sigma_{-i}, [s_i])$ for some $\sigma \in M(S)$, then $\mu_i(C_i) \leq \varepsilon \cdot \mu_i(D_i)$.*

The following result is immediately obtained from Proposition 2, establishing the existence of at least one proper equilibrium with support in any given finely tenable block.

Corollary 1. *Every finely tenable block contains the support of a proper equilibrium.*

Proof: By the Bolzano-Weierstrass theorem, every sequence from a nonempty compact set has a convergent subsequence with limit in the set. Given T finely tenable, let τ^* be such a limit point of a sequence $\langle \tau^k \rangle_{k \in \mathbb{N}}$ of Nash equilibria $\tau^k \in M(F)$ of consideration-set games G^{μ_k} where each μ_k is an ε_k-proper type distribution on T and $\varepsilon_k \to 0$. Let σ^k and $\sigma^* \in M(S)$ be the projections of τ^k and τ^* in G. By construction, σ^* is a proper equilibrium of G. Moreover, σ^* has support in T, because $\mu^k(T) \to 1$ and thus $\forall s_i \in S_i$,

$$\sigma_i^k(s_i) = \sum_{C_i \in \mathcal{C}(S_i)} \mu_i^k(C_i) \cdot \tau_{i|C_i}^k(s_i) \quad \to \quad \sigma_i^*(s_i) = \tau_{i|T_i}^*(s_i),$$

so $\sigma_i^*(s_i) = 0$ if $s_i \notin T_i$. **Q.E.D.**

Remark 4. *The above machinery provides a behavioral micro foundation for the concept of proper equilibrium. For by Proposition 2 as applied to $T = S$, all limit points (as $\varepsilon \to 0$) to projections of sequences of Nash equilibria in the associated consideration-set games are proper equilibria.*

In the light of Proposition 2 and Corollary 1 it is natural to require properness when defining settledness with respect to finely tenable blocks:

Definition 9. *A **finely settled equilibrium** is any proper equilibrium that has support in some minimal finely tenable block.*

We call any equilibrium that is both finely and coarsely tenable *fully settled*.

Proposition 3. *Every finite game has at least one fully settled equilibrium.*

Proof: Let T be any minimal coarsely tenable block (the existence of which follows from the finiteness of S). Then T is also finely tenable. If T is not a minimal finely tenable block, T will contain such a block (again since S is finite). According to the above corollary, there exist a proper equilibrium with support in that subblock.[16] **Q.E.D.**

[16] Since curb $\Rightarrow$ absorbing $\Rightarrow$ coarsely tenable $\Rightarrow$ finely tenable, every finite game in fact admits a fully settled equilibrium with support in a minimal absorbing block that is a subset of a minimal curb block.

In the following elaboration of game (1) for $\alpha = \beta = \gamma = \delta = 1$, the only coarsely tenable block is the whole strategy space S. By contrast, there are smaller finely tenable blocks. Moreover, the finely settled equilibria correspond to the two strict equilibria of the original game (1).

Example 2. *Reconsider the extensive-form game in Example 1. If one would replace the* $(0,0)$ *end-node by another zero-sum subgame, like the first zero-sum subgame, the purely reduced normal form (with abstract labeling of pure strategies) would be*

Game 2:

	ax_2	ay_2	bx_2	by_2
ax_1	$1,1$	$1,1$	$2,-2$	$-2,2$
ay_1	$1,1$	$1,1$	$-2,2$	$2,-2$
bx_1	$2,-2$	$-2,2$	$1,1$	$1,1$
by_1	$-2,2$	$2,-2$	$1,1$	$1,1$

Also this elaboration of game (1) has three Nash equilibrium components:

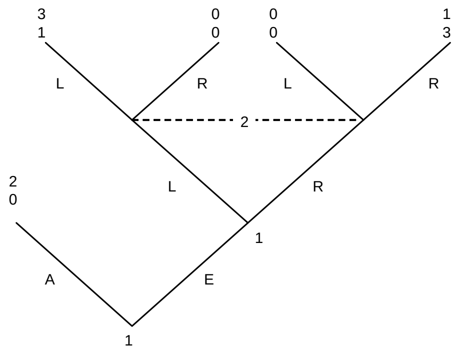
$$A = \{\sigma = (p[ax_1] + (1-p)[ay_1]\ ,\ q[ax_2] + (1-q)[ax_2]))\ \text{for}\ p,q \in (1/4, 3/4)\}$$

$$B = \{\sigma = (p[bx_1] + (1-p)[by_1]\ ,\ q[bx_2] + (1-q)[bx_2]))\ \text{for}\ p,q \in (1/4, 3/4)\}$$

and $C = \{\sigma^m\}$, *where* σ^m *is uniform randomization over each strategy set. The proper equilibria are* $\sigma^a = (\frac{1}{2}[ax_1] + \frac{1}{2}[ay_1], \frac{1}{2}[ax_2] + \frac{1}{2}[ay_2]) \in A$, $\sigma^b = (\frac{1}{2}[bx_1] + \frac{1}{2}[by_1], \frac{1}{2}[bx_2] + \frac{1}{2}[by_2]) \in B$, *and* σ^m. *The only curb, absorbing or coarsely tenable block is the whole pure-strategy space* S, *so all Nash equilibria are persistent and coarsely settled. However,* $T^a = \{ax_1, ay_1\} \times \{ax_2, ay_2\}$ *and* $T^b = \{bx_1, by_1\} \times \{bx_2, by_2\}$ *are finely tenable blocks by similar arguments to those given in Example 5. The game thus has only two finely, indeed fully, settled equilibria,* σ^a *and* σ^b, *corresponding to, and behaviorally indistinguishable from, the two strict equilibria of the original game (1).*

The finely settled equilibria in the preceding example are also coarsely settled. The next example shows that a finely settled equilibrium need not be coarsely settled.

Example 3. *Consider a version of the battle-of-the-sexes game where player 1 has an outside option:*



Its purely reduced normal form (with A representing the two payoff-equivalent strategies AL and AR) is

Game 3:

	L	R
EL	$3,1$	$0,0$
ER	$0,0$	$1,3$
A	$2,0$	$2,0$

This normal-form game has two Nash equilibrium components, the singleton set $T^* = \{EL\} \times \{L\}$, consisting of the strict pure equilibrium $s^* = (EL, L)$, the "forward-induction" solution, and a continuum component in which player 1 plays A for sure while player 2 plays R with probability at least $1/3$. The strict equilibrium s^* is fully settled. Another proper equilibrium of this game is $s^o = (A, R)$. (To see this, note that for all $\varepsilon > 0$ small enough, $\sigma_1^\varepsilon = (\varepsilon^2, \varepsilon, 1 - \varepsilon - \varepsilon^2)$ and $\sigma_2^\varepsilon = (\varepsilon, 1 - \varepsilon)$ make up an ε-proper strategy profile σ^ε.) What about its supporting block, $T^o = \{A\} \times \{R\}$? Clearly, T^o is not coarsely tenable, since strategy L is the unique best reply if there is a positive probability that 1 plays EL, which indeed is the case under some type distributions that attach arbitrary little, but positive probability to strategies outside the block T^o. However, T^o is finely tenable. To see this, let μ be an ε-proper type distribution on T^o and let $\tau \in M(F)$ be any Nash equilibrium of G^μ. Then $\tau_2^\mu(L) < \varepsilon$, so for $\varepsilon > 0$ small enough $\tau_1^\mu(EL) \leq \mu_1(\{EL\}) \leq \varepsilon \cdot \mu_1(\{EL, ER\}) \leq \tau_1^\mu(ER)$, which implies that A and R are best replies to τ^μ. In sum, s^* is fully settled while s^o is finely but not coarsely settled. Note that s^o corresponds to the sequential equilibrium of the extensive-form game in which play of (R, R) is expected in the battle-of-sexes subgame.

We proceed to establish that coarsely and finely tenable blocks, and thus also coarsely and finely settled equilibria, are generically equivalent.

6. Generic normal-form games

The concept of regular equilibrium was introduced by Harsanyi (1973) and slightly modified by van Damme (1991), who defined a Nash equilibrium of a finite normal-form game to be regular if the Jacobian, associated with a certain system of equations closely related to those characterizing Nash equilibrium, is non-singular (op. cit. Definition 2.5.1).

Definition 10. *A game $G = \langle N, S, u \rangle$ is* **hyper-regular** *if, for every block $T \subseteq S$, all Nash equilibria of the associated block game G_T are regular in the sense of van Damme (1991).*

In a well-defined sense, almost all normal-form games are hyper-regular:

Lemma 1. *For any (finite) set of players N and (finite) sets of strategies S_i for each player $i \in N$, the set of payoff functions u in $\mathbb{R}^{|N|\cdot|S|}$ such that $G = \langle N, S, u \rangle$ is not hyper-regular is contained in a closed set of Lebesgue measure zero in $\mathbb{R}^{|N|\cdot|S|}$.*

Proof: The property of regularity of a block game G_T depends only on the payoffs on T, and this property will fail only for payoff functions in a closed set of Lebesgue measure zero (van Damme, 1991, Theorem 2.6.1). There are only finitely many blocks $T \subseteq S$, and the union of finitely many such sets is still a closed set of measure zero. **Q.E.D.**

Proposition 4. *If a game $G = \langle N, S, u \rangle$ is hyper-regular, then any block T is finely tenable if and only if it is coarsely tenable. Any equilibrium of G is finely settled if and only if it is coarsely settled.*

Proof: We first establish that for a hyper-regular game there cannot exist any Nash equilibrium τ of any block game G_T such that a player i has an alternative $s_i \in S_i \backslash T_i$ with $u_i(\tau_{-i}, [s_i]) = u_i(\tau)$. If this equality would hold, and if we added s_i to T_i (obtaining $T'_i = T_i \cup \{s_i\}$), then we would obtain a block game $G_{T'}$ in which τ would still be a Nash equilibrium but, having an alternative best reply in T' that gets zero probability in τ, τ would not be quasi-strict in this new block game $G_{T'}$. By hyper-regularity of G, τ is a regular equilibrium of $G_{T'}$ and hence τ is quasi-strict (Corollary 2.5.3 in van Damme, 1991), a contradiction.[17] So if τ is a Nash equilibrium of some block game G_T, then either $u_i(\tau_{-i}, [s_i]) > u_i(\tau)$ or $u_i(\tau_{-i}, [s_i]) < u_i(\tau)$. The first of these inequalities, for any i and s_i, would imply that T is not coarsely tenable. The second inequality, for all i and s_i, would imply that T is coarsely tenable. Thus, in the given hyper-regular game G, a block T is coarsely tenable if and only if $u_i(\tau_{-i}, [s_i]) < u_i(\tau)$ for all i and $s_i \in S_i \backslash T_i$, at all equilibria τ of the block game. Coarsely tenable blocks are always finely tenable, so it remains to

[17] A *quasi-strict equilibrium* (Harsanyi, 1973) is any Nash equilibrium in which all players use all their pure best replies.

prove that, for our hyper-regular game G, any block T that is not coarsely tenable is not finely tenable.

In order to establish this, consider any Nash equilibrium τ of any block game G_T. The payoff function of G_T can be viewed as a vector u in $\mathbb{R}^{|N|\cdot|T|}$. By hyper-regularity of G, the equilibrium τ is regular, and thus also strongly stable, in G_T (van Damme, 1991, Definition 2.4.4 and Theorem 2.5.5). This means that there is some open neighborhood V of $\tau \in M(T)$ and some open neighborhood U of $u \in \mathbb{R}^{|N|\cdot|T|}$ such that, for any perturbation of G_T that has a payoff function $\tilde{u}$ in U, we obtain a game $\tilde{G}_T = \langle N, T, \tilde{u}\rangle$ that has exactly one equilibrium $\tilde{\tau}$ in V, and this equilibrium depends continuously on the payoff function $\tilde{u}$.

Now let's think about a consideration-set game G^μ. Let ρ be a partial behavior-strategy profile in the extensive form of G^μ that defines a mixed strategy $\rho_{i|C_i} \in \Delta(C_i)$ for every player i, the player's local strategy at that information set, and for every consideration set C_i other than T_i. Let $B(T)$ be the set of all such partial behavior-strategy profiles. When ρ defines the behavior of players at all consideration sets other than those of T, then the only question remaining in G^μ is what each player i would do when considering T_i, which will happen with probability at least $1-\varepsilon$. So with any given ε, the consideration-set game G^μ becomes a perturbation of G_T, and its payoff function in $\mathbb{R}^{|N|\cdot|T|}$ will be in the open set U for all $\varepsilon > 0$ sufficiently small, given ρ. In fact, there exists an $\bar{\varepsilon} > 0$ such that $\tilde{u} \in U$ for all ρ. Now, given any $\varepsilon \in (0, \bar{\varepsilon})$, consider the correspondence that sends any profile $\rho \in B(T)$ to each player i's (non-empty, compact and convex) best local replies at every consideration set $C_i \neq T_i$ to the ρ and $\tilde{\tau}$ strategies, where $\tilde{\tau}$ is the (continuously defined) equilibrium in V for this perturbation of G_T. This correspondence is upper hemi-continuous in ρ, so, by Kakutani's fixed-point theorem, for any such ε, there exists a fixed point ρ^*. This fixed-point ρ^*, together with its corresponding $\tilde{\tau}$ at T, will constitute an equilibrium of the consideration-set game G^μ. The projection of this equilibrium to $M(S)$ will be ε-proper with respect to the block T (along the lines given in Section 5) and these strategy profiles converge to the given block-game equilibrium τ as $\varepsilon \to 0$. But then this sequence would yield a contradiction of T being tenable if we had $u_i(\tau_{-i}, [s_i]) > u_i(\tau)$ for some player i and some strategy $s_i \in S_i \backslash T_i$. Thus, if T is not coarsely tenable, then T is not finely tenable either. So for a hyper-regular game, a block is coarsely tenable if and only if it is finely tenable.

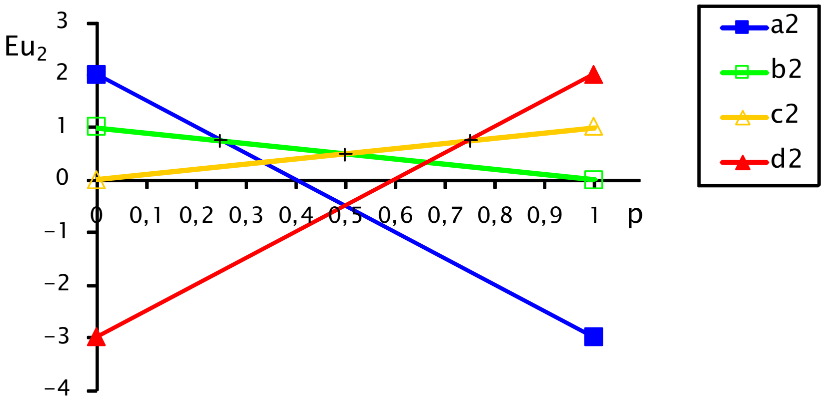
Since all regular equilibria are proper (van Damme, 1991, Theorems 2.5.5, 2.4.7, 2.3.8), an equilibrium in a hyper-regular game is coarsely settled if and only if it is finely settled. **Q.E.D.**

The game in the introduction shows that there are generic games in which the set of settled equilibria is strictly smaller than the set of Nash (perfect, proper) equilibria. The game in the next example, taken from Table 7 in Myerson (1996), has an open neighborhood (in the space of 2×4 normal-form games) in which there is always a Nash equilibrium which is persistent but not settled. This shows that settled equilibrium is not generically equivalent with Nash or persistent equilibrium.

Example 4. *Consider the game*

$$\text{Game 4:} \qquad \begin{array}{ccccc} & a_2 & b_2 & c_2 & d_2 \\ a_1 & 0,2 & 1,1 & 0,0 & 1,-3 \\ b_1 & 1,-3 & 0,0 & 1,1 & 0,2 \end{array} \tag{6}$$

and note that the block game over the "middle block" $T^{bc} = \{a_1, b_1\} \times \{b_2, c_2\}$ is identical with game (1) for $\alpha = \beta = \gamma = \delta = 1$. The diagram below shows the payoffs to player 2's pure strategies as functions of the probability p by which player 1 uses her first pure strategy.



This game has three Nash equilibria, all mixed. In each equilibrium, player 1 uses both her pure strategies while player 2 uses only two of his four pure strategies; either the two left-most, $\{a_2, b_2\}$, the two middle ones, $\{b_2, c_2\}$, or the two right-most,$\{c_2, d_2\}$, each equilibrium corresponding to a kink in the upper envelope of the payoff lines in the above diagram.

This game is hyper-regular. This follows from Theorem 7.4 in Jansen (1981) (see also Theorem 3.4.5 in van Damme, 1991), according to which a Nash equilibrium of a finite two-player game is regular if and only if it is essential and quasi-strict.[18] *It is not difficult to verify that all block equilibria, of all blocks in this game, have both properties. Moreover, the game has only one curb block, the whole set S, and it*

[18] An *essential equilibrium* (Wu and Jiang, 1962) is any Nash equilibrium such that every nearby game, in terms of payoffs, has some nearby Nash equilibrium.

has only one absorbing retract, the whole set $M(S)$. Hence, all three equilibria are persistent. However, the "middle" equilibrium is not settled.

More exactly, the three Nash equilibria of this game are

$$\sigma^{ab} = \left(\frac{3}{4}[a_1] + \frac{1}{4}[b_1], \frac{1}{2}[a_2] + \frac{1}{2}[b_2]\right)$$

$$\sigma^{bc} = \left(\frac{1}{2}[a_1] + \frac{1}{2}[b_1], \frac{1}{2}[b_2] + \frac{1}{2}[c_2]\right)$$

$$\sigma^{cd} = \left(\frac{1}{4}[a_1] + \frac{3}{4}[b_1], \frac{1}{2}[c_2] + \frac{1}{2}[d_2]\right)$$

Consider first the "middle" block $T^{bc} = \{a_1, b_1\} \times \{b_2, c_2\}$, the support of σ^{bc}. The associated block contains, in addition to σ^{bc}, two (strict pure) block-game equilibria, each, however, with better replies outside the block. Hence, by Proposition 1, this block is not coarsely tenable. Arguably, if T^{bc} became the conventional block played in a population, play might drift towards one of these strict block equilibria, which would induce a movement out of the block, towards a better reply, and thereby destabilize the block. By contrast, the supports of each of the two other equilibria, the "side" blocks $T^{ab} = \{a_1, b_1\} \times \{a_2, b_2\}$ and $T^{cd} = \{a_1, b_1\} \times \{c_2, d_2\}$, do not contain any other block equilibria and are coarsely tenable. The only coarsely tenable block that contains σ^{bc} is S, which, however, is not minimal. Hence, while all three equilibria are persistent, only σ^{ab} and σ^{cd} are coarsely settled. These claims hold for an open set of payoff perturbations of the game. Thus, the property of being coarsely settled is not generically equivalent to persistence.

The two minimal coarsely tenable blocks, T^{ab} and T^{cd}, are, a fortiori, also finely tenable. Since they contain no other finely tenable block, they are minimal and hence σ^{ab} and σ^{cd} are also finely settled. Is the middle block T^{bc} finely tenable? As we will see, this is not the case although T^{bc} is the support of a proper equilibrium, σ^{bc}. First, to see that σ^{bc} is proper, let $\sigma_1^{\varepsilon} = (1/2, 1/2)$ and $\sigma_2^{\varepsilon} = (\varepsilon, 1/2 - \varepsilon, 1/2 - \varepsilon, \varepsilon)$. Clearly σ^{ε} is ε-proper for all $\varepsilon \in (0, 1/2)$, and $\sigma^{\varepsilon} \to \sigma^{bc}$ as $\varepsilon \to 0$. Second, to see that T^{bc} is not finely tenable, let $\varepsilon > 0$ and let μ be as in Remark 2 , with $\delta_2(b_2) = \varepsilon$, $\delta_2(d_2) = \varepsilon \cdot \delta_2(c_2) = \varepsilon^2 \cdot \delta_2(a_2)$ and $\|\delta_i\| < \varepsilon/(1+\varepsilon)$ for both players i. Then μ is ε-proper on T^{bc} for all $\varepsilon > 0$. However, for all $\varepsilon > 0$ sufficiently small and all Nash equilibria $\tau \in M(F)$ of G^{μ} we have

$$u_2(\tau_1^{\mu}, \tau_{2|T_2}) < u_2(\tau_1^{\mu}, [a_2]), \tag{7}$$

so T^{bc} is not finely tenable. In sum: σ^{ab} and σ^{cd} are the only fully settled equilibria of this game.

7. MORE EXAMPLES

Suppose that game (1) was enlarged by letting each player's mixed Nash-equilibrium strategy be represented as a new pure strategy. As the following example shows, this would not affect the collection of minimal tenable blocks. Hence, the rejection of the mixed equilibrium in that game does not depend on the fact that it is mixed *per se*.

Example 5. *Let $\alpha = \beta = \gamma = \delta = 1$, and consider*

$$\text{Game 5:} \quad \begin{array}{cccc} & a_2 & b_2 & c_2 \\ a_1 & 1,1 & 0,0 & \lambda,\lambda \\ b_1 & 0,0 & 1,1 & \lambda,\lambda \\ c_1 & \lambda,\lambda & \lambda,\lambda & \lambda,\lambda \end{array},$$

for $\lambda < 1$. For $\lambda = 1/2$, the new pure strategy c_i is payoff equivalent with the mixed Nash equilibrium strategy $\sigma_i^* = \frac{1}{2}[a_i] + \frac{1}{2}[b_i]$, for $i = 1, 2$, in game (1). For $\lambda = 1/2$, this 3×3-game has infinitely many Nash equilibria; the two strict equilibria (a_1, a_2) and (b_1, b_2), the pure equilibrium (c_1, c_2), and a continuum of mixed equilibria where each player i randomizes arbitrarily between σ_i^* and c_i (thus also including (c_1, c_2) as an extreme point). The two strict equilibria are of course fully settled. Moreover, their supports are the only minimal tenable blocks (and this holds for an open set of payoffs around $\alpha = \beta = \gamma = \delta = 1$). In particular, the singleton block $T^c = \{c_1\} \times \{c_2\}$ is not coarsely tenable, and this holds for all $\lambda < 1$. The reason is that for arbitrarily small $\varepsilon > 0$ there are type distributions μ such that $\mu_i(\{c_i\}) > 1 - \varepsilon$ for $i = 1, 2$, under which c_i is not a best reply to the projection of any Nash equilibrium in the associated consideration-set game G^μ. For example, for each player i let $\mu_i(\{c_i\}) = 1 - \varepsilon/2$ and $\mu_i(\{a_i\}) = \varepsilon/2$ (thus all other consideration sets have probability zero). Let τ be any Nash equilibrium of G^μ. Then $\tau_i^\mu(a_i) = \varepsilon/2$ and $\tau_i^\mu(b_i) = 0$, so $\max_{t_i \in T_i^c} u_i(\tau_{-i}^\mu, [t_i]) = \lambda$ while (for any $\lambda < 1$)

$$\max_{s_i \in S_i} u_i(\tau_{-i}^\mu, [s_i]) = 1 \cdot \varepsilon/2 + \lambda \cdot (1 - \varepsilon/2) > \lambda.$$

Established refinements, such as Kohlberg-Mertens stability, are known to have little bite in sender-receiver games. By contrast, settledness effectively discards arguably implausible equilibria in such games. We illustrate this by way of a simple example due to Balkenborg, Hofbauer and Kuzmics (2014).[19]

Example 6. *Consider a sender-receiver game in which there are two equally likely states of nature, $\omega = A$ and $\omega = B$. Player 1, the sender, observes the state of nature and sends one of two messages, a or b, to player 2. Having received 1's message, 2*

[19] See also the analysis of related issues in Gordon (2011).

takes one of two actions, α or β. Hence, each player has four pure strategies. Assume that both players receive payoff 2 if action α (β) is taken in state A (B), and otherwise both players receive payoff zero. The normal form of this game is

Game 6:

	$\alpha\alpha$	$\alpha\beta$	$\beta\alpha$	$\beta\beta$
aa	$1,1$	$1,1$	$1,1$	$1,1$
ab	$1,1$	$2,2$	$0,0$	$1,1$
ba	$1,1$	$0,0$	$2,2$	$1,1$
bb	$1,1$	$1,1$	$1,1$	$1,1$

Any strategy pair that assigns equal probability to the two middle strategies is a Nash equilibrium: if $\sigma_1 = (p, q, q, r) \in \Delta(S_1)$ and $\sigma_2 = (p', q', q', r') \in \Delta(S_1)$, then σ is a Nash equilibrium. As pointed out by Balkenborg et al. (2014), every Nash equilibrium σ of this kind, viewed as a singleton set, is strategically stable in the sense of Kohlberg and Mertens (1986). However, only the two pure and strict equilibria $s^* = (ab, \alpha\beta)$ and $s^{**} = (ba, \alpha\beta)$ ("taking the right action in each state") seem reasonable as predictions of how people will play this game, especially if they had some familiarity with this or similar interactions. For the same reasons as given in the preceding example, only these two equilibria are settled.

Voting games are well-known to exhibit a plethora of Nash equilibria, many of which seem unreasonable and yet resist standard refinements such as perfection and strategic stability. The next example suggests that the present solutions have good cutting power, beyond that of perfection and strategic stability, and that it selects natural equilibria. The examples the so-called *Duverger's law*, which asserts that the plurality rule for selecting the winner of elections favors the two-party (or two-candidate) system (see Riker, 1982).

Example 7. *There are three candidates, call them 1, 2, and 3, of whom candidates 1 and 2 are ideologically similar, say on the left of the political spectrum, and then candidate 3 is on the right side of the spectrum. In the election, each voter must choose to vote for one candidate, and the candidate with the most votes wins; a tie for the most would be resolved by random selection among those in the tie. There are seven voters. Each voter can be characterized by his or her vector $v = (v_1, v_2, v_3)$ of utilities for each of the three candidates winning the election. Three voters are rightist partisans of candidate 3 and have utility vector $(0, 0, 1)$. The other four voters are leftists and can be called 1A with utility vector $(4, 1, 0)$, 1B with utility vector $(3, 2, 0)$, 2A with utility vector $(1, 4, 0)$, and 2B with utility vector $(2, 3, 0)$. In any proper equilibrium, all three rightists always vote for the rightist candidate 3.*

We find two settled equilibria that have two serious candidates getting all the votes, as predicted by Duverger's law for such plurality elections. In one of these *settled equilibria, all four leftist voters, $\{1A, 1B, 2A, 2B\}$, vote for candidate 1, who then wins the election with probability 1. In the other of these settled equilibria, all four leftist voters vote for candidate 2, who then wins. In either of these equilibria, if one leftist voter deviated to vote for the other leftist candidate, there would be a tie, and the rightist candidate would win with probability 1/2. Each of these Duvergerian equilibria is a strict pure equilibrium, and its support is a minimal tenable block.*

But we can also find a third equilibrium, one in which the leftist voters split their votes among candidates 1 and 2. In this equilibrium, voter 1A votes for candidate 1 for sure, voter 1B randomizes, voting for candidate 1 with probability 0.6 and for candidate 2 with probability 0.4, voter 2A votes for candidate 2 for sure, and voter 2B randomizes, voting for candidate 2 with probability 0.6 and for candidate 1 with probability 0.4. In this equilibrium, the rightist candidate 3 wins with probability 0.76 while each of the leftist candidates has probability 0.12 of winning. This equilibrium violates Duverger's law, and it is not settled. The support of this equilibrium is not tenable, because its block game would also have pure-strategy equilibria in which voters 1B and 2B both vote for the same leftist candidate (candidate 1 or 2), which would not be an equilibrium of the original game because it would make voter 1A or 2A switch over to also vote for the same leftist candidate. Thus, any tenable block that includes the support of this mixed equilibrium must also include the support of both Duvergerian equilibria, and so it is not minimal among all tenable blocks. However, the mixed equilibrium is regular and hence proper.

Our final example shows that, unlike minimal curb blocks, minimal tenable blocks may overlap.

Example 8. *Consider*

Game 8:

	a_2	b_2	c_2
a_1	$3,1$	$1,3$	$0,0$
b_1	$1,3$	$3,1$	$1,3$
c_1	$0,0$	$1,3$	$3,1$

This game has three Nash equilibria: σ^{ab}, in which each player randomizes uniformly across his or her two first pure strategies, σ^{bc}, in which they randomize uniformly across their last two pure strategies, and the totally mixed

$$\sigma^m = \left(\frac{2}{9}[a_1] + \frac{5}{9}[b_1] + \frac{2}{9}[c_1], \frac{2}{5}[a_2] + \frac{1}{5}[b_2] + \frac{2}{5}[c_2]\right)$$

The supports of σ^{ab} and σ^{bc} are blocks that both contain (b_1, b_2). These blocks are not absorbing, since for certain mixed-strategy profiles near the profile that puts unit *probability on b_1 and b_2, either player 1 or 2 has no best reply in the block. However, they are minimal coarsely tenable. The only absorbing block is the full strategy space S, so all equilibria are persistent, while only σ^{ab} and σ^{bc} are coarsely, indeed, fully settled.*

8. Conclusion

This paper has focused on an assumption that, in culturally familiar games, people will develop social conventions that simplify the game by excluding some strategies from normal consideration. We feel that such an assumption has substantial realism, not only because of the prevalence of social conventions but also because of cognitive limitations. In games with very large strategy spaces, such as chess, it falls outside the bounds on human cognition to consider one's whole set of pure strategies. Our assumption, to allow for the possibility that a player may ignore some strategies that are feasible in the actual game, leads us to analyze games that might not be common knowledge among the players. But we have assumed also that players can break free from such conventions and explore other strategies in the game. Thus, our concepts of tenable blocks have been defined as conditions for a conventional simplification to justify players' understanding that they have no reason to consider unconventional alternatives as long as others are unlikely to do so.

To formalize these concepts, we have analyzed consideration-set games in which players may randomly and independently consider any nonempty subset of their actual strategy set in the game. We defined a coarsely tenable block as a convention such that there could never be any advantage for any player to consider any unconventional strategy in any equilibrium of any consideration-set game in which the probability of every player considering the given conventional block is sufficiently close to one. This seemed a good basic definition of tenability, but the coarsely settled equilibria that we found in minimal coarsely tenable blocks sometimes failed to exclude some equilibria that seemed unreasonable to us, and so we developed a concept of fine tenability that would admit smaller tenable blocks.

Our concept of fine tenability was derived by analyzing a smaller class of consideration set games around any given conventional block, those in which any unconventional player would be much more likely to see more of the strategies that are feasible in the actual game. This restriction implies some rationality of behavior when players deviate from the convention and brings the present approach, except for its focus on conventions, close to the standard game-theoretic approach where players always consider their full strategy sets. However, other classes of consideration-set games around a conventional block might also be worth studying. Any other restriction on the consideration-set games (within those admitted in our definition of coarse tenability) could yield an alternative concept of "weak tenability" which in turn could be used to define an alternative concepts of settled equilibria in the minimal weakly tenable blocks. We would look forward to future research on these ideas.

By way of examples, we showed the nature and power of these solution concepts. In particular, the rejection of the mixed equilibrium in the coordination game in the introduction was shown to hold even when one or both of the zero-payoff outcomes was replaced by a zero-sum game, and also when the mixed Nash equilibrium strategy was added as a third pure strategy. In other examples we showed that minimal tenable blocks and settled equilibria make well-behaved and sharp predictions in some games where standard refinements usually fail. The difference was particularly stark in a signaling game and in a voting game. In the battle-of-the-sexes game with an outside-option, the forward-induction solution was seen to be the only fully settled equilibrium, while the "outside option" equilibrium component contained another finely (but not coarsely) settled equilibrium. Yet other examples showed that the present approach makes sharper predictions than curb and persistence in an open set of games. A final example demonstrated that there are games in which tenable blocks overlap.

The present approach also suggests other avenues for further research, where one is to apply the current solutions to well-known (finite) games that represent important interactions in economics, political science and other social and behavior sciences. Do the solutions suggested here match up with what we know or believe about the likely outcomes in such interactions? For example, standard refinements usually have little cutting power in voting games, and yet such games usually have a plethora of arguably unreasonable Nash equilibria. We conjecture that the present machinery might have a lot of cutting power in such games.[20] A second avenue would be to study the solutions' predictive power in controlled laboratory experiments. Will human subjects in the lab, under random rematching and with some opportunity for social learning, tend towards minimal tenable blocks and settled equilibria? A third avenue could be to explore connections between our solutions and explicit models of population dynamics. There is a handful such models in the economics literature. Some of these have been shown to converge to minimal curb sets, see Young (1993, 1998), Hurkens (1995) and Sanchirico (1996). It is also known from the literature on dynamic learning and evolution in games (see e.g. Nachbar, 1990, and Weibull, 1995) that if such a process meets certain regularity conditions, and if it converges, then the limit point will be a Nash equilibrium. In such dynamic population models, will settled equilibria and minimal tenable blocks be good predictors?

[20] Laslier and Van den Straaten (2004) show that while perfection may have little cutting power, "true perfecting" (see Kalai and Samet, 1984) effectively eliminates implausible equilibria. The present machinery might lead to similar conclusions as for true perfection, since tenability effectively requires robustness to a wide range of type distributions, much in the same way as true perfection requires robustness to a wide range of strategy perturbations.

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